Hands-on: Integration rules for Calculus

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Indefinite integral

If $f(x)$ is the derivative of function $f(x)$ then $\int f(x)dx = F(x) + C$

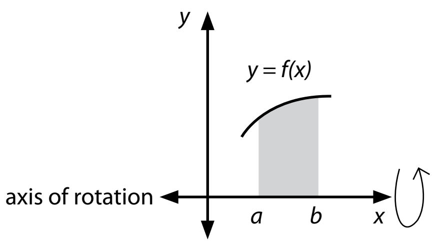
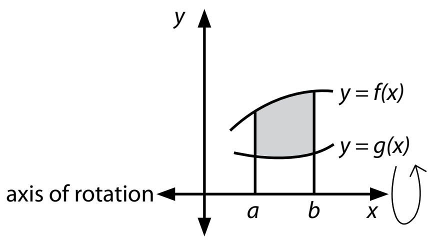
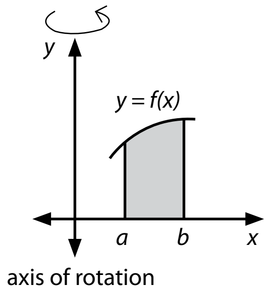
Definite integral

$$\int_a^b f(x)dx = \left[\int f(x)dx\right]_a^b = [F(x)]_a^b = F(b) - F(a)$$

Area

Area bounded by $y = f(x)$, the x-axis and the lines $x = a$ and $x = b$	Area bounded by $x = f(y)$, the y-axis and the lines $y = c$ and $y = d$	Area bounded by $y = f(x)$ and $y = g(x)$ and the lines $x = a$ and $x = b$
$A_x = \int_a^b f(x)dx$	$A_y = \int_c^d f(y)dy$	$A_x = \int_a^b [f(x) - g(x)]dx$

Volume

Region bounded by $y = f(x)$, the x-axis and the lines $x = a$ and $x = b$ rotated about the x-axis (Disc method)	Region bounded by $y = f(x)$ and $y = g(x)$ and the lines $x = a$ and $x = b$ rotated about the x-axis (Washer method)	Region bounded by $y = f(x)$, the x-axis and the lines $x = a$ and $x = b$ rotated about the y-axis (Shell method)
		
$V_x = \pi\int_a^b [f(x)]^2 dx$	$V_x = \pi\int_a^b ([f(x)]^2 - [g(x)]^2)dx$	$V_y = 2\pi\int_a^b xf(x)dx$

Simpson's rule

$$\int_a^b f(x)dx \approx \frac{\Delta x}{3}[f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + \dots + 2f(x_{n-2}) + 4f(x_{n-1}) + f(x_n)]$$

where n is even and $\Delta x = \frac{b-a}{n}$

Integration by partial fractions

$$\int \frac{(ax+b)}{((x-\alpha)(x-\beta))\,dx} = \int \frac{A}{(x-\alpha)\,dx} + \int \frac{B}{(x-\beta)\,dx}$$

Integration by parts

$$\int u\,dv = uv - \int v\,du$$

Rule	Function	Integral
Sum/difference	$f(x) \pm g(x)$	$F(x) \pm G(x) + C$
Product of constant and function	$kf(x)$	$kF(x) + C$
Constant	k	$kx + C$
Power	x^n	$\frac{1}{n+1}x^{n+1} + C, n \neq -1$
Reciprocal	x^{-1}	$\ln x + C$
Generalised power	$f'(x)(f(x))^n$	$\frac{1}{n+1}(f(x))^{n+1} + \mathrm{C}, n \neq -1$
Exponential (Base e)	$f'(x)e^{f(x)}$	$e^{f(x)} + C$
Natural logarithm	$\frac{f'(x)}{f(x)}$	$\ln f(x) + C$

TRIGONOMETRIC AND HYPERBOLIC FORMS			
Function	**Integral**	**Function**	**Integral**
$\sin x$	$-\cos x + C$	$\sec^2 x$	$\tan x + C$
$\cos x$	$\sin x + C$	$\mathrm{cosec}^2 x$	$-\cot x + C$
$\tan x$	$\ln(\sec x) + C$	$\sec x \tan x$	$\sec x + C$
$\mathrm{cosec}\, x$	$\ln(\mathrm{cosec}\, x - \cot x) + C$	$\mathrm{cosec}\, x \cot x$	$-\mathrm{cosec}\, x + C$
$\sec x$	$\ln(\sec x + \tan x) + C$	$\sinh x$	$\cosh x + C$
$\cot x$	$\ln(\sin x) + C$	$\cosh x$	$\sinh x + C$

FORMS INVOLVING SQUARES			
Function	**Integral**	**Function**	**Integral**
$\frac{1}{x^2 + a^2}$	$\frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + C$	$\frac{1}{\sqrt{a^2 - x^2}}$	$\sin^{-1}\left(\frac{x}{a}\right) + C$
$\frac{1}{x^2 - a^2}$	$\frac{1}{2a}\ln\left(\frac{x-a}{x+a}\right) + C$	$\frac{1}{\sqrt{x^2 \pm a^2}}$	$\mathrm{In}\,(x + \sqrt{x^2 \pm a^2}) + C$

Arc length

$$s = \int_a^b \sqrt{1 + [f'(x)]^2}dx$$

Average value

$$A(f) = \frac{1}{b-a}\int_a^b f(x)dx$$

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